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MINIMUM ENERGY TIME-OPTIMAL CONTROL FOR SATURATING SAMPLED-DATA SYSTEMS

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Abstract

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The paper considers the time-optimal regulator problem for sampled-data control systems with input saturation and minimal control energy. It is shown that of the infinite number of different control sequences that can, in general, give time-optimal control when the input is constrained by saturation, there is only one among them that also minimizes the control energy. A design procedure for both saturating and non-saturating systems is formulated from the theory and an example demonstrates the method. The example also brings out an interesting point when minimum fuel and minimum energy control sequences are compared. The method is restricted neither by the nature of the poles nor by the order of the system.

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Summary

The paper develops the theory of, and a design procedure for, a minimal energy time-optimal regulator for sampled-data control systems with input saturation. The time-optimal regulator problem for sampled-data control systems with input saturation has received much attention recently. It has the property that there are, in general, an infinite number of control sequences, $u(k)$, $k = 1, 2, \ldots, N$, which take a given initial state to a desired final state in the minimum time, N sampling periods. More recently there have been attempts to choose from these sequences one that minimizes not only the time but also satisfies some other criterion. Such a criterion might be minimizing the 'fuel' F, where

$$F = \sum_{k=1}^{N} |u(k)|,$$

or the 'energy' E, where

$$E = \sum_{k=1}^{N} u(k)^2 .$$

The minimum fuel problem has been solved by linear programming (1) in general, and by an analytical method for a restricted case (2). The minimum energy problem is solved here. Using the state space approach this paper shows that the control sequence which minimizes the energy is unique and the method is restricted neither by the nature of the poles nor by the order of the system.

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The theory is developed, for a fixed number of sampling periods, in two parts. The first part considers the case where the input is not constrained by saturation. The second part considers the input to be constrained by saturation. In this case the well known time-optimal set ${}_N$ (3) is divided into two separate sets, M_N and P_N. If the initial state lies within the set M_N the problem is essentially the same as if the input were unconstrained, whereas if it lies in P_N the saturation constraint makes the theory more involved.

The design procedure given is very simple for the case without saturation and for initial states in the set M_N. The procedure is more involved for states in P_N, but a systematic procedure is developed which is no more difficult than recent approaches to the time-optimal problem alone. The design procedure for a fixed integer N is extended in the paper to solve the minimum time problem.

An example is given which demonstrates the procedure and shows the improvement in energy consumption over the time-optimal sequences obtained previously by others. The example also brings out an interesting point: Minimizing the energy E can also minimize the fuel F, and furthermore the solution to the minimum fuel problem is not unique in general.

The method compares favorably with the possible use of nonlinear programming techniques in that considerable insight is gained into the real nature of the minimum energy problem for sampled-data control systems. Thus, for particular systems sub-optimal strategies would be evident and could possibly be used to trade optimum performance with, for example, particularly straightforward hardware implementation. The method also shows that the z-transform would have been little use for this problem.

Fifteen references have been listed to give a complete introduction and accompaniment to the paper.

<u>References for the Summary</u>

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